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Multiple Detection Probabilistic Data Association Filter for Multistatic Target Tracking

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***Abstract*—A standard assumption in most tracking algorithms, like the Probabilistic Data Association (PDA) filter, Multiple Hypothesis Tracker (MHT) or the Multiframe Assignment Tracker (MFA), is that a target is detected at most once in a frame of data used for association. This one-to-one assumption is essential for correct measurement-to-track associations. When this assumption is violated, the above algorithms treat the extra detections as random clutter. When multiple detections from the same target fall within the association gate, the PDA filter tries to apportion the association probabilities, but with the fundamental assumption only one of them is correct. The MFA and the MHT algorithms try to spawn multiple tracks to handle the additional measurements from the same target, assuming at most one measurement came from each target. Both of these approaches have undesirable side effects since they ignore the possibility of multiple detections from the same target in a scan of data. Such multiple detection situations occur in multistatic tracking problems.**

In this paper, we proposed a new Multiple Detection Probabilistic Data Association (MD-PDA) filter for tracking a target when more than one target originated measurement may exist within the validation gate. In the proposed MD-PDA, combinatorial association events are formed to handle the possibility of multiple measurements from the same target. Modified association probabilities are calculated with the explicit assumption of multiple detections. Simulations are presented to demonstrate the effectiveness of the algorithm on a single target tracking problem in clutter. Extensions to handle multiple targets using the Joint PDA, MHT and MFA approaches are under development.

Keywords: target tracking in clutter, multistatic tracking, data association, probabilistic data association

I. Introduction

Tracking a target in clutter where it is unknown which of the received set of measurements is originated from target has been one of the most challenging issues. In the literature, several techniques have been proposed for this data association problem to identify target originated measurement from a clutter [2], [4], [6], [12], [13]. Non-Bayesian, one-to-one matching, hard decision oriented data association solutions are the Nearest Neighbor Filter (NNF) and Strongest Neighbor Filter (SNF) [2]. As their names imply, the NNF updates a track with the measurement closest to the predicted measurement among the validated measurements while the SNF associates the measurement with the strongest intensity.

The aforementioned data association techniques perform well in terms of computation and estimation accuracy in a scenario where the target return is very strong and the false alarm rate is low. With degraded observability and dense clutter such approaches begin to fall short. Under such conditions, a more practical approach to deal with measurement origin uncertainty is applying Bayesian association techniques. One of Bayesian association approach is the sub-optimal Probabilistic Data Association (PDA) filter [3], [4], [5]. The PDA estimator avoids a hard association decision by updating a track with a set of measurements and their corresponding weights. In the PDA estimator the weight corresponding to each validated measurement is calculated by assessing all possible measurement-to-track combinations. As a result, the weight assigned to a given measurement inside the validation gate is the probability that it came from that track.

The PDA estimator is appropriate for single target tracking problem. The Multiple Target Tracking (MTT) brings more challenge to the data association as it has to be determined which measurement belongs to which target besides identifying target originated measurement from a clutter. Thus, the PDA has to be extended to handling multiple targets, resulting in the joint PDA (JPDA) algorithm that can handle tracking multiple targets by evaluating the joint probabilities among the tracks and the measurements [13]. In addition, the interacting multiple model is usually integrated to handle target maneuvers in IMM-JPDA [7]. Furthermore, multiple scan JPDA and multipattern algorithm have been developed as an extension to the PDA [9], [15].

An alternative optimal Bayesian approach to MTT is the multiple hypothesis tracker (MHT) [6]. MHT handles the multitarget tracking problem by forming multiple hypotheses and evaluating the likelihood that there is a target in a given sequence of measurements. Although it is an optimal approach, within few steps it will become computationally infeasible. Hypothesis pruning techniques can be applied to the MHT approach for practical problems at the expense of optimality [10]. Another approach to the MTT problem is Multiframe Assignment Tracker (MFA) [8] that models the measurement-to-track association as a constrained optimiza-

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14. ABSTRACT

A standard assumption in most tracking algorithms like the Probabilistic Data Association (PDA) filter, Multiple Hypothesis Tracker (MHT) or the Multiframe Assignment Tracker (MFA), is that a target is detected at most once in a frame of data used for association. This one-to-one assumption is essential for correct measurement-to-track associations. When this assumption is violated, the above algorithms treat the extra detections as random clutter. When multiple detections from the same target fall within the association gate, the PDA filter tries to apportion the association probabilities, but with the fundamental assumption only one of them is correct. The MFA and the MHT algorithms try to spawn multiple tracks to handle the additional measurements from the same target, assuming at most one measurement came from each target. Both of these approaches have undesirable side effects since they ignore the possibility of multiple detections from the same target in a scan of data. Such multiple detection situations occur in multistatic tracking problems. In this paper, we proposed a new Multiple Detection Probabilistic Data Association (MD-PDA) filter for tracking a target when more than one target originated measurement may exist within the validation gate. In the proposed MD-PDA, combinatorial association events are formed to handle the possibility of multiple measurements from the same target. Modified association probabilities are calculated with the explicit assumption of multiple detections. Simulations are presented to demonstrate the effectiveness of the algorithm on a single target tracking problem in clutter. Extensions to handle multiple targets using the Joint PDA, MHT and MFA approaches are under development.

15. SUBJECT TERMS

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tion problem.

The assumption in most tracking algorithms, like the aforementioned PDA, JIPDA, MHT or the Multiframe Assignment Tracker (MFA), is that a target is detected at most once in a frame of data used for association [6], [8], [12], [14]. This one-to-one assumption is essential for correct measurement-to-track associations. When this assumption is violated, for example, when a target is detected more than once per scan, the above algorithms treat the extra detections as random clutter or tend to spawn multiple tracks for the same target.

Such multiple-detection situations occur in multistatic tracking problems. When multiple detections from the same target fall within the association gate, the PDA estimator as well as its multitarget version, JIPDA, try to apportion the association probabilities, but with the fundamental assumption only one of them is correct. The MFA and the MHT algorithms try to spawn multiple tracks to handle the additional measurements from the same target, due to the basic assumption at most one measurement came from each target. Both of these approaches have undesirable side effects since they ignore the possibility of multiple detections from the same target in a scan of data. A mechanism that accounts for the possibility of multiple detections from the same target should be developed so that all useful information from the received measurements about the target state is extracted.

In order to rectify the above short coming, a new Multiple Detection Probabilistic Data Association (MD-PDA) is presented in this paper. In the proposed MD-PDA, combinatorial association events are formed to handle the possibility of multiple measurements from the same target. Multiple association events are formed by creating φ out of m combinations of multiple measurements to track assignment, where φ is the number of target originated measurements and m is the total number of measurements in the validation gate. The number of target originated measurements can be used as a known prior to determine the probability of detection condition on φ, which is $P_{D\varphi}$. For each association events, the probabilities will be calculated and based on the probabilities a measurement or set of measurements will be associated to a target.

The enhanced capability of the fusing all the information from target originated measurements in the proposed MD-PDA will manifest if the sensor data contains multiple detection in a single frame. If the target is detected only once per frame the MD-PDA filter cannot perform better than original PDA. Simulation is done by generating multiple detection measurements from a single target observed in a clutter. MD-PDA performance is compared with the original PDA. Performance evaluation results show the effectiveness, with respect to estimation accuracy, of the proposed algorithm as a result of taking the possibility of multiple detections into account. However, the algorithm tends to take more time due to increased number of association events. Similar extensions to handle duplicate detections in the presence of multiple targets using the JPDA, MHT and MFA approaches are under development.

The remainder of the paper is organized as follows. Section II discusses the multiple detection pattern. Models for combinatorial events in the presence of multiple detection is presented in this section. The new MD-PDA filter is presented in Section III where theoretical developments of MD-PDA are discussed. Simulation results is presented Section V, which is based on a single target tracking in clutter. Finally, conclusion is drawn in Section VI.

II. Multiple Detection Pattern

When multiple detections from the same target fall within the association gate, a measurement or set of measurements might be associated to a target. Data association uncertainty with multiple detection can be resolved by generating a multiple detection pattern. The multiple detection pattern will consider all possible events for measurement-to-track association.

Assume that the targets state evolves according to a dynamic equation driven by process noise

$$x(k+1) = F(k)x(k) + w(k) \tag{1}$$

and the measurement equation by

$$z(k) = H(k)x(k) + v(k) \tag{2}$$

where $x(k)$ represents target state, $F(k)$ is the system transition matrix and $H(k)$ is the measurement matrix. $w(k)$ and $v(k)$ are white and independent system and measurement noise respectively.

For m number of measurement inside the validation gate φ out of m association events are evaluated while φ runs from one to the maximum number of target originated measurements. This association event represent all possible events from single target originated measurement to all of the measurements are target originated. For example, as depicted in Figure 1, there are four measurements $(z_1(k), z_2(k), z_3(k), z_4(k))$ in the data frame. Out of four measurements, three of them $(z_1(k), z_2(k), z_3(k))$ are inside the validation gate. Combinatorial association events are created only for those measurements that fall inside the validation gate. The maximum number of target originated measurement is assumed to be $\varphi_{max} = 3$. Thus the possible events are:

- none of the measurements is target orignated
  - $\varphi = 0$, $n_\varphi = 1$
- one of the measurements is target originated
  - $\varphi = 1$, $n_\varphi = \text{comb}(3,1) = 1,2,3$
  - 3 measurement-to-track association events
  - $z_1(k)$ or $z_2(k)$ or $z_3(k)$ is originated from a target

$$z_{1,1}(k) = z_1(k) \tag{3}$$
$$z_{1,2}(k) = z_2(k) \tag{4}$$
$$z_{1,3}(k) = z_3(k) \tag{5}$$

- two of the measurements are target originated
  - $\varphi = 2$, $n_\varphi = \text{comb}(3,2) = 1,2,3$
  - 3 measurements-to-track association events

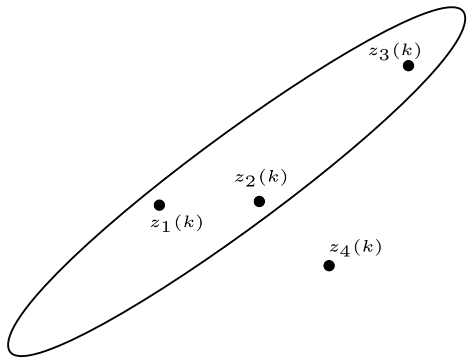

Figure 1. Validation Gate

- $z_1(k), z_2(k)$ or $z_1(k), z_3(k)$ or $z_2(k), z_3(k)$ are originated form a target

$$z_{2,1}(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} \tag{6}$$

$$z_{2,2}(k) = \begin{bmatrix} z_1(k) \\ z_3(k) \end{bmatrix} \tag{7}$$

$$z_{2,3}(k) = \begin{bmatrix} z_2(k) \\ z_3(k) \end{bmatrix} \tag{8}$$

- all of the measurements are target originated
  - $\varphi = 3$, $n_\varphi = \text{comb}(3,3) = 1$
  - 1 measurements-to-track association event
  - $z_1(k), z_2(k), z_3(k)$ are originated form a target

$$z_{3,1}(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \\ z_3(k) \end{bmatrix} \tag{9}$$

Accordingly the measurement equation (2) for the (φ, n_φ) event becomes

$$z_{\varphi,n_\varphi}(k) = \begin{bmatrix} H_1(k) \\ \vdots \\ H_\varphi(k) \end{bmatrix} x(k) + \begin{bmatrix} v_1(k) \\ \vdots \\ v_\varphi(k) \end{bmatrix} \tag{10}$$

III. Multiple Detections Probability Data Association (MD-PDA)

The approach of the standard PDAF is to calculate the association probabilities for each validated measurement that falls in a gate around the predicted measurement at the current time to the target of interest [2]. When two of the measurements are target originated, the algorithm apportion the total weight to both of them, with the assumption that only one of them is target originated. This is not efficient approach especially when there are false alarms in the validation gate. The MD-PDA algorithm calculates the probability that each set of measurements, rather than a single measurement, is attributable to the target of interest. The set of measurements candidate for evaluation are generated from multiple detection pattern discussed above. This probabilistic (Bayesian) information based on the candidate set of measurements is used in a tracking filter, that updates the target states.

A. Assumptions

The following assumptions are made for the MD-PDA filter

- Among the validated measurements, a measurement or set of measurements can originate from a target.
- The target detections occur independently over time with known probabilities.
- Clutter is uniform/Possion distributed within the measurement validation gate.
- There is only one target of interest whose state evolves according to a dynamic equation driven by process noise as stated in (1).
- Track has been initiated.

At each time k, the MD-PDA algorithm runs through the following steps.

B. Gating

A validation gate is set up for each time step to determine the candidate measurements for association. The validation gate is an ellipse [2] given by

$$V(k,\gamma) = \{z : [z - \hat{z}(k|k-1)]' S(k)^{-1} [z - \hat{z}(k|k-1)] \leqslant \gamma\} \tag{11}$$

where γ is the gate threshold and $S(k)$ is the innovation covariance corresponding to the measurement given by

$$S(k) = H(k)P(k|k-1)H(k)' + R(k) \tag{12}$$

The volume is thus given by

$$\begin{aligned} V(k) &= c_{n_z}|\gamma(S(k)|^{1/2} && (13) \\ &= c_{n_z}\gamma^{\frac{n_z}{2}}|(S(k)|^{1/2} && (14) \end{aligned}$$

where n_z is the dimension of the measurement and the coefficient c_{n_z} is the volume of the n_z-dimensional unit hypersphere ($c_1 = 2, c_2 = \pi, c_3 = 4\pi/3$, etc.).

C. MD-PDA approach

The latest set of validated measurements is denoted as

$$Z(k) = \{z_i(k)\}_{i=1}^{m(k)} \tag{15}$$

where $z_i(k)$ is the i^{th} validated measurement and $m(k)$ is the number of measurements in the validation region at time k. The cumulative set of measurements up to time step k is

$$Z^k = \{Z(j)\}_{j=1}^{k} \tag{16}$$

For the association events

$$\theta_{\varphi,n_\varphi}(k) = \begin{cases} (\varphi \text{ out of } m(k) \text{ are target originated }) \\ \qquad n_\varphi = 1, ..., c_{\varphi m}(k) \\ (\text{none of the measurements is target originated}) \\ \qquad n_\varphi = 0 \end{cases} \tag{17}$$

where $c_{\varphi m}(k)$ is φ combinations out of $m(k)$ measurements given by

$$c_{\varphi m}(k) = \begin{pmatrix} m(k) \\ \varphi \end{pmatrix} \tag{18}$$

The number of association events grow very fast for $\varphi > 2$. The expected number of target originated measurement can be used as a priori to reduce the number of events. Applying the total probability theorem with respect to the above events, the conditional mean of the state at time k is given as

$$\begin{aligned} \hat{x}(k|k) &= E(x(k)|Z^k) \\ &= \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} E(x(k)|\theta_{\varphi,n_\varphi}(k), Z^k) p(\theta_{\varphi,n_\varphi}(k)|Z^k) \\ &= \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} \hat{x}_{\varphi,n_\varphi}(k|k) \beta_{\varphi,n_\varphi}(k) \end{aligned} \tag{19}$$

where $\hat{x}_{\varphi,n_\varphi}(k|k)$ is the updated state which is conditioned on the event that the (φ, n_φ) set of measurements are correct. Here the association probability, $\beta_{\varphi,n_\varphi}(k)$, is the conditional provability of the event.

$$\beta_{\varphi,n_\varphi}(k) \propto p(\theta_{\varphi,n_\varphi}(k)|Z^k) \tag{20}$$

The estimate conditioned on n_φ^{th} combination of φ measurements being correct is

$$\hat{x}_{\varphi,n_\varphi}(k|k) = \hat{x}(k|k-1) + W_{\varphi,n_\varphi}(k)\nu_{\varphi,n_\varphi}(k)$$

where the corresponding innovation is

$$\nu_{\varphi,n_\varphi}(k) = \begin{bmatrix} (z(k) - \hat{z}(k|k-1))' \\ \vdots \\ (z(k) - \hat{z}(k|k-1))' \end{bmatrix} \tag{21}$$

and the Kalman gain $W(k)$ is

$$W_{\varphi,n_\varphi}(k) = P(k|k-1)H_{\varphi,n_\varphi}(k)' S_{\varphi,n_\varphi}(k)^{-1} \tag{22}$$

Here

$$S_{\varphi,n_\varphi}(k) = H_{\varphi,n}(k)P(k|k-1)H_{\varphi,n_\varphi}(k)' + R_{\varphi,n_\varphi}(k) \tag{23}$$

$$H_{\varphi,n_\varphi}(k) = \begin{bmatrix} H(k) \\ \vdots \\ H(k) \end{bmatrix} \tag{24}$$

$$R_{\varphi,n_\varphi}(k) = \begin{bmatrix} R(k) & 0 & \dots & 0 \\ 0 & R(k) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & R(k) \end{bmatrix} \tag{25}$$

D. State and Covariance Update

The state update equation is given by

$$\begin{aligned} \hat{x}(k|k) &= \hat{x}(k|k-1) \\ &\quad + W_{\varphi,n_\varphi}(k) \sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} \beta_{\varphi,n_\varphi}(k)\nu_{\varphi,n_\varphi}(k) \end{aligned} \tag{26}$$

and the covariance associated with the updated state is

$$P(k|k) = \beta_0(k)P(k|k-1) + (1-\beta_0(k))P^c(k|k) + \tilde{P}(k) \tag{27}$$

where the covariance of the state updated with the correct measurement is

$$P^c(k|k) = P(k|k-1) - W_{\varphi,n_\varphi}(k)S_{\varphi,n_\varphi}(k)W_{\varphi,n_\varphi}(k)' \tag{28}$$

and the spread of innovation term, $\tilde{P}(k)$, is given in (29).

E. MD Association Probabilities

There will be $m(k)$ validated measurements at time k. Among these validated measurements one, two or φ number of measurements can be target originated. Multiple detection association probabilities are evaluated by probabilistic inference which is made on

- number of measurements in the validation region, $m(k)$
- number of target originated measurements, φ
- location of measurements

which is expressed as

$$\beta_{\varphi,n_\varphi}(k) = p(\theta_{\varphi,n_\varphi}(k)|Z^k, m(k), \varphi, Z^{k-1}) \tag{30}$$

Applying Bayes' theorem

$$\begin{aligned} \beta_{\varphi,n_\varphi}(k) = &\frac{1}{c} p(Z^k|\theta_{\varphi,n_\varphi}(k), m(k), \varphi, Z^{k-1}) \\ &\times p(\theta_{\varphi,n_\varphi}(k)|m(k), \varphi, Z^{k-1}) \end{aligned} \tag{31}$$

The first term in (31) refers to the joint density of the pdf of the correct measurement is given in (32) where P_G is the factor that accounts for restricting the normal density to the validation gate. The second term in (31) is the probability of the association events conditioned only on $m(k)$ and φ.

$$\begin{aligned} \gamma_{\varphi,n_\varphi}(k) &= p(\theta_{\varphi,n_\varphi}(k)|m(k), \varphi, Z^{k-1}) \\ &= p(\theta_{\varphi,n_\varphi}(k)|m(k), \varphi) \end{aligned} \tag{33}$$

where the probability $\gamma_{\varphi,n_\varphi}$ evaluates the event $\theta_{\varphi,n_\varphi}$ conditioned on the total number of validated measurement $\mathcal{M} = m$. Here $\mathcal{M}$ denotes the random variable and m its realization [2].

$$\begin{aligned} \gamma_{\varphi,n_\varphi}(k) &= p(\theta_{\varphi,n_\varphi}|\mathcal{M} = m(k), \varphi) \\ &= p(\theta_{\varphi,n_\varphi}|\Psi = m(k) - \varphi, \mathcal{M} = m(k)) \\ &\quad \times p(\Psi = m(k) - \varphi|\mathcal{M} = m(k)) \\ &\quad + p(\theta_{\varphi,n_\varphi}|\Psi = m(k), \mathcal{M} = m(k)) \\ &\quad \times p(\Psi = m(k)|\mathcal{M} = m(k)) \end{aligned} \tag{34}$$

where Ψ is the number of false measurements. For φ target originated measurements, Ψ must be either $m(k) - \varphi$ or $m(k)$.

$$\tilde{P}(k) \triangleq W_{\varphi,n_\varphi}(k) \left[\sum_{\varphi=0}^{m(k)} \sum_{n_\varphi=1}^{c_{\varphi m}(k)} \beta_{\varphi,n_\varphi}(k)\nu_{\varphi,n_\varphi}(k)\nu_{\varphi,n_\varphi}(k)' - \nu(k)\nu(k)' \right] W_{\varphi,n_\varphi}(k)' \tag{29}$$

$$p(Z^k|\theta_{\varphi,n_\varphi}(k), m(k), \varphi, Z^{k-1}) = \begin{cases} \frac{1}{P_G} \times V(k)^{-m(k)+1} \mathcal{N}(\nu_{\varphi,n_\varphi}(k); 0, S(k)) & n_\varphi = 1, ..., c_{\varphi m}(k) \\ V(k)^{-m(k)} & n_\varphi = 0 \end{cases} \tag{32}$$

$$\gamma_{\varphi,n_\varphi}(k) = \begin{cases} \frac{1}{m(k)} \times p(\Psi = m(k) - \varphi | \mathcal{M} = m(k)) \\ \qquad\qquad n_\varphi = 1, ..., c_{\varphi m}(k) \\ p(\Psi = m(k) | \mathcal{M} = m(k)) \\ \qquad\qquad n_\varphi = 0 \end{cases} \tag{35}$$

where

$$\begin{aligned} p(\Psi &= m(k) - \varphi | \mathcal{M} = m(k)) \\ &= \frac{p(\mathcal{M} = m(k) | \Psi = m(k) - \varphi) p(\Psi = m(k) - \varphi)}{p(\mathcal{M} = m(k))} \\ &= \frac{P_{D\varphi} P_G \mu(m(k) - \varphi)}{p(\mathcal{M} = m(k))} \end{aligned} \tag{36}$$

and

$$\begin{aligned} p(\Psi &= m(k) | \mathcal{M} = m(k)) \\ &= \frac{p(\mathcal{M} = m(k) | \Psi = m(k)) p(\Psi = m(k))}{p(\mathcal{M} = m(k))} \\ &= \frac{(1 - P_D P_G)\mu(m(k))}{p(\mathcal{M} = m(k))} \end{aligned} \tag{37}$$

$P_{D\varphi}$ is the probability of detecting a target φ times per scan. The total probability of detection P_D will become the superposition of detection probabilities of $P_{D\varphi}$. Also, $P_{D\varphi}P_G$ is the probability that the target has been detected and φ measurements originated from it are inside the gate and $(1 - P_D P_G)$ is the probability that the measurements in the gate are false alarms. Thus

$$\begin{aligned} p(\mathcal{M} = m(k)) = \sum_{\varphi=1}^{m(k)} & P_{D\varphi} P_G \mu(m(k) - \varphi) \\ & + (1 - P_D P_G)\mu(m(k)) \end{aligned} \tag{38}$$

Substituting (38) in (36) and (37), the result in (35)

$$\gamma_{\varphi,n_\varphi}(k) = \begin{cases} \frac{1}{m(k)} \frac{P_{D\varphi} P_G \mu(m(k)-\varphi)}{\sum_{\varphi=1}^{m(k)} P_{D\varphi} P_G \mu(m(k)-\varphi) + (1 - P_D P_G)\mu(m(k))} \\ \qquad\qquad n_\varphi = 1, ..., c_{\varphi m}(k) \\ \frac{(1 - P_D P_G)\mu(m(k))}{\sum_{\varphi=1}^{m(k)} P_{D\varphi} P_G \mu(m(k)-\varphi) + (1 - P_D P_G)\mu(m(k))} \\ \qquad\qquad n_\varphi = 0 \end{cases} \tag{39}$$

For the probability mass function of the number of false measurements, $\mu(m(k))$, a Poisson or diffused prior model can be used in the volume $V(k)$ (see section III-B).

- Poisson model (parametric MD-PDA):

$$\mu(m(k)) = e^{-\lambda V(k)} \frac{(\lambda V(k))^{m(k)}}{m(k)!} \tag{40}$$

  where λ is spacial density.
- Diffuse prior model (non-parametric MD-PDA):

$$\mu(m(k)) = \mu(m(k) - \varphi) = K \tag{41}$$

  where K is a constant.

Finally, by substituting (39) in (31), the association probabilities for a measurement or φ measurements set can be computed with parametric (40) or non-parametric (41) false measurements model of MD-PDA.

IV. Simulations

A surveillance region covering an area of 1000 m long and 1500 m wide is used to test potential advantages of the multiple target originated measurement approach. Measurements are generated by a 2D radar with the following properties:

- $P_{D1} = 0.05$ is probability of detecting a target once per scan of the measurement data
- $P_{D2} = 0.9$ is probability of detecting a target twice per scan of the measurement data
- $P_D = P_{D1} + P_{D2} = 0.95$ which is total probability of detecting a target in a scan of the measurement data (i.e, P_D used for PDA)
- $P_{FA} = 10^{-5}/m^2$ with Poisson distribution

A single target starts from origin and moving with constant speed of 15 m/s parallel to the x-axis is considered. Target initialization is done using two point target initialization method. The scan interval (sampling period) is 1 s and it consists of 50 scans. For the MD-PDA the probabilities of detections used are P_{D1} and P_{D2} while $P_D = P_{D1} + P_{D2}$ is total probability of detecting a target used in PDA.

Figure 2 shows the Root Mean Square Error (RMSE) for position that demonstrates the improved performance of multiple detection approach over the classic probability data association. As PDA tends the apportion the weight among the target originated measurements, MD-PDA assigns the weight to measurement set, rather than a single measurement, that are

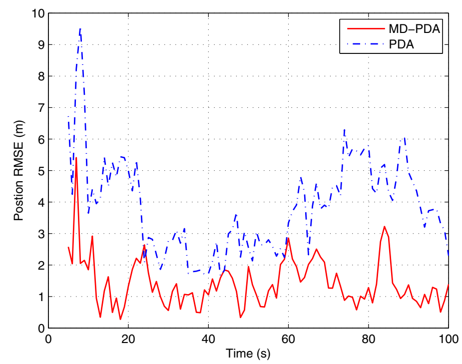

Figure 2. Position RMSE evaluation for MD-PDA vs. PDA

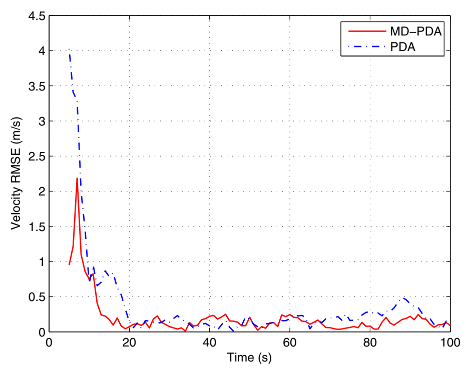

Figure 3. Velocity RMSE evaluation for MD-PDA vs. PDA

originated form a target. The velocity RMSE evaluation result is shown in Figure 3.

The performance evaluation result which is based on 1000 MonteCarlo Runs is presented in Table I. With respect to Position and Velocity RMSE the MD-PDA performs better than PDA. This is because unlike PDA, the MD-PDA updates the filter with the set of measurements that are originated form a target. Due to more association events evaluation the MD-PDA takes longer time than PDA.

Table I
PERFORMANCE EVALUATION (MD-PDA VS. PDA)

Performance Matrix	MD-PDA	PDA
Position RMSE	1.62 m	2.83 m
Velocity RMSE	0.35 m/s	0.74 m/s
Average Latency	0.11 s	0.08 s

V. CONCLUSIONS

In this paper a new Multiple Detection Probabilistic Data Association (MD-PDA) filter was proposed. The algorithm is designed for tracking a target while receiving multiple detections form the same target within the same scan of measurements. When multiple detections from the same target fall within the association gate, the standard PDA filter returns degraded estimation results due to violation of one measurement per scan assumption. In the proposed MD-PDA, combinatorial association events are formed to handle the possibility of multiple measurements from the same target. Modified association probabilities are calculated with the explicit assumption of multiple detections. Experimental results show the effectiveness of the proposed algorithm. Similar extensions to handle multiple targets using the JPDA filter, MHT and MFA tracker are under development. Also further work has to be done to initialize targets with multiple detections.

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